TRANSIENT HEAT TRANSFER FROM FLUID FLOWING IN BURIED PIPE

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ABSTRACT

The paper presents numerical investigation for transient heat transfer from fluid flowing inside a buried horizontal pipe. The top surface of the soil, in which the pipe is buried is kept at a temperature lower than the fluid temperature. The effect of the type of the soil taking into account its porosity on the transient heat loss from the fluid flowing inside the pipe is studied assuming laminar flow. The solution technique used in modeling this problem is based on finite element formulation in two dimensions. The generality of the approach developed here with respect to arbitrary boundary conditions and changes in media properties provides a useful means of handling problems of heat transfer in similar complex systems. The validity of the proposed scheme has been demonstrated by comparing the model results with the available analytical solution for a simplified but related problem. Applications of this investigation are found in steam lines, water and oil distribution lines, ..etc.

1-INTRODUCTION:

In recent years, the problem of heat loss from fluids flowing through pipes buried in a porous medium have become subject of numerous investigation because of its considerable practical interest. This problem arises in steam lines of power plants, industrial and agricultural water distribution lines, buried electrical cables, oil and gas transmission lines, and in the storage piles of nuclear waste.

The early studies that considered the problem of buried pipes and cables have been based on idealized cases. Approximate and exact steady state heat loss calculations are available in the literature for pipes with simplified boundary conditions such as isothermal surfaces [1], and using exact solution via bicylindrical coordinates or uniform heat flux surfaces [2]. The approximate solutions given by Eckert and Drake, [3] are based on the superposition of infinite-line heat source and sink solutions. The steady-state heat transfer with a uniform heat transfer coefficient along the pipe wall was modeled by Bau and Sadhal [4]. In their investigation Bau and Sadhal considered the case of laminar flow with linear temperature variation along the pipe axis and analytical solutions were obtained via bicylindrical coordinates. The finite thermal conductivity of the pipe wall was neglected. Himasekhar and Bau [5] took into account the porous nature of the soil in which the pipe was buried, but an isothermal pipe surface was assumed.

The thermal resistance to heat transfer between the surface of a buried pipe and ground level has been calculated for the case where both the pipe surface and the and the ground surface are isothermal, through the use superposition of line source and sink solutions [6,7].

Some approximate time-dependent analytical solutions were obtained by Ioffe, [8]. Martin and Sadhal [9], obtained an analytical transient temperature distribution due to buried cylinder which was treated as a heat source. The effect of boundary conditions on the thermal resistance, where the ground surface remains isothermal but the pipe surface has a uniform flux boundary had been examined Lebedev et al.[10]. Recently, a time and space finite difference model had been used by Negiz et al. [11] to obtain the temperature distribution in the pipe and the soil. A special case for certain configuration of the pipe buried in the soil was considered.

From the above review it can be concluded that there still some aspects regarding the influencing parameters of the heat loss from fluid flowing through pipes buried in porous media need more investigation.

The objective of the work presented here is to develop a general model capable of handling the most important heat transfer parameters and practical variables in the field of buried pipe lines in order to achieve the best working conditions. Therefore, the effect of buried depth, flow characteristics the type of the soil and the conduction in the pipe wall on the heat loss from the fluid flowing through the pipe are studied. The problem is formulated as being transient in two dimensions. Three different soils encompassing a wide range of properties are examined taking the effect of porosity into consideration.

2-MATHEMATICAL ANALYSIS:

The problem considered here is a pipe with inner radius R_i, outer radius R_o and buried a distance h beneath the surface of a semi-infinite porous medium as depicted schematically in Fig. (1). Initially the porous medium, the pipe wall, and the fluid inside the pipe are all at uniform temperature T_i. Suddenly a new temperature T_b , lower than the initial temperature T_i is introduced at the top surface of the medium. Here the situation where water is pumped through a buried pipe from a constant temperature reservoir T_o, is considered. At the entrance plane of the pipe all media are kept at T_o all the times. Also the flow is assumed to be laminar inside the pipe and the end effects are ignored.

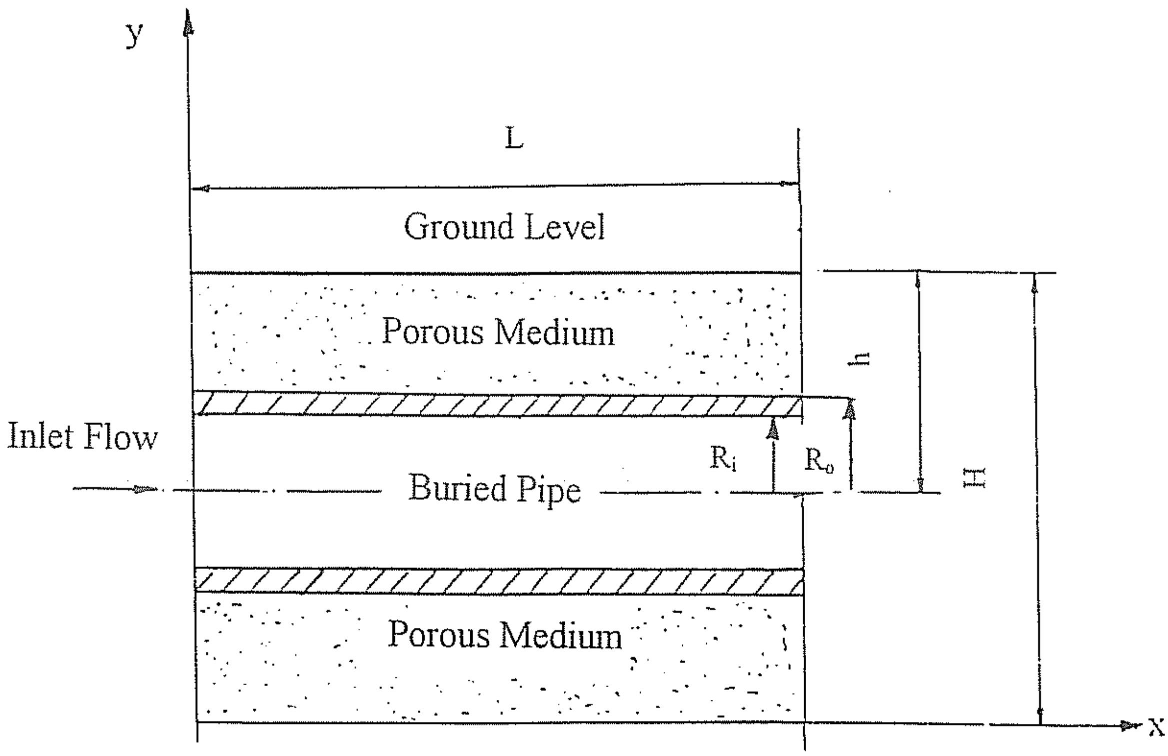

Fig. (1) Schematic diagram of the problem

2.1-Basic Equations:

The general form of energy equation in two dimensions with axial fluid flow and transient condition can be written as follows :

$$k\frac{\partial^2 T}{\partial x^2}+k\frac{\partial^2 T}{\partial y^2}=\rho c\frac{\partial T}{\partial t}+\rho c u\frac{\partial T}{\partial x} \qquad (1)$$

(1) Inside the pipe

$$k = k_f$$
$$\rho = \rho_f$$
$$c = c_f$$
$$u = 2\,U\,[1 - (y/R_i)^2]$$

(2) In the pipe wall

$$k = k_p$$
$$\rho = \rho_p$$
$$c = c_p$$
$$u = 0.0$$

(3) In the soil

$$k = k_{es} = (1-\Phi)\,k_s + \Phi\,k_a$$
$$\rho = \rho_{es} = (1-\Phi)\,\rho_s + \Phi\,\rho_a$$
$$c = c_{es} = (1-\Phi)\,c_s + \Phi\,cp_a$$
$$u = 0.0$$

The corresponding initial and boundary conditions are:

Initial condition:

at $t = 0$ $\quad T = T_i \quad$ in all media (2)

Boundary conditions:

$T = T_b \quad$ at $y = H \quad$ for all x and t (3)

$T = T_o \quad$ at $x = 0 \quad$ for all y, t and all media (4)

$\frac{\partial T}{\partial x} = 0 \quad$ at $x = L \quad$ for all y, t and all media (5)

$\frac{\partial T}{\partial y} = 0 \quad$ at $y = 0 \quad$ for all x and t (6)

The above system of equations with the initial and boundary conditions stated for the different media made the analytical solution impossible for such type of proplems. The alernative solution in this case is rely on the numerical techniques.

3-NUMERICAL SOLUTION:

The energy Equation (1) is solved using the Galerkin based finite element method [12,13,14]. The objective of the finite element is to reduce the system of governing equations into a discretized set of algebric equations. The procedure begins with the division of the continuum region of interest into a number of simply shaped regions called elements.

The Finite Element Formulation

The temperature in an element T^e as shown in Fig. (2) can be represented in terms of nodal temperature T_m by simple polynomial:

$$T^e = \sum_{m=1}^{3} N_m T_m \tag{7}$$

where;

$$N_1 = \frac{1}{2A}\,(a_1 + b_1\,x + c_1\,y)$$
$$N_2 = \frac{1}{2A}\,(a_2 + b_2\,x + c_2\,y)$$
$$N_3 = \frac{1}{2A}\,(a_3 + b_3\,x + c_3\,y)$$

A = area of the triangle 123

$a_1 = x_2\,y_3 - x_3y_2$
$b_1 = y_2 - y_3$
$c_1 = x_3 - x_2$

$a_2 = x_3\,y_1 - x_1y_3$
$b_2 = y_3 - y_1$
$c_2 = x_1 - x_3$

$a_3 = x_1\,y_2 - x_2y_1$
$b_3 = y_1 - y_2$
$c_3 = x_2 - x_1$

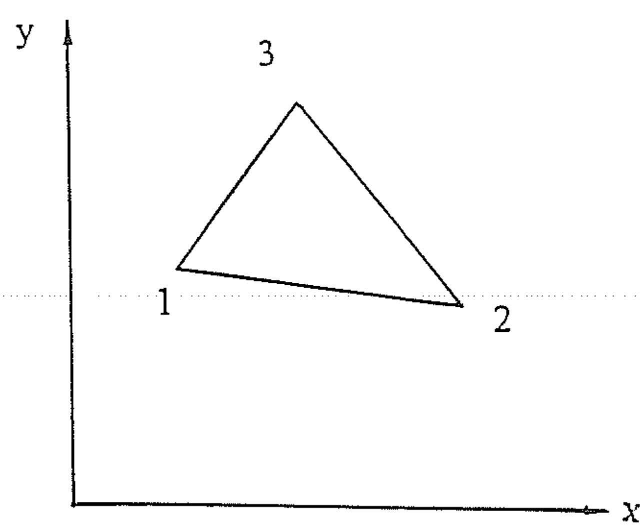

Fig. (2) Linear triangular element

The interpolation function $[N_1 \quad N_2 \quad N_3]$ in Eq.(7) is derived from an assumption of linear variation of temperature in the element. The approximate expression of T^e is substituted into Eq. (1) and the global error is minimized using the above interpolation function N_i (i = 1, 2, 3) as a weighting function. The solution of Eq. (1) that satisfies the boundary conditions, Eq.(2) to Eq.(6), can be written after weighted integration over the domain Ω^e and the application of Green's theorem, in the equivalent matrix form as:

$$[K]\{T\} + [K_c]\{\dot{T}\} = \{F\}$$

where,

$$[K] = [K_I] + [K_{II}]$$

$$[K_I] = \sum_{e=1}^{E} \int_{\Omega^e} \left(k \frac{\partial[N]^T}{\partial x} \cdot \frac{\partial[N]}{\partial x} + k \frac{\partial[N]^T}{\partial y} \cdot \frac{\partial[N]}{\partial y}\right) d\Omega$$

$$[K_{II}] = \sum_{e=1}^{E} \int_{\Omega^e} \rho c u [N]^T \frac{\partial[N]}{\partial x} d\Omega$$

$$[K_c] = \sum_{e=1}^{E} \int_{\Omega^e} \rho c [N]^T [N] d\Omega$$

$$\{F\} = \sum_{e=1}^{E} \int_{\Gamma^e} k [N]^T \frac{\partial T}{\partial n} d\Gamma$$

where;

E = total number of elements, Ω bounded domain, Γ domain boundary

The resulting finite element equations have been solved through a computer program written here in FORTRAN language.

4-ANALYTICAL SOLUTION FOR SIMPLIFIED PROBLEM:

The validity of the proposed model is made by comparing its results with those available for heat conduction in a uniform rectangular region as a simplified problem, Fig. (3). The energy equation is written in dimensionless form as:

$$\alpha \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) = \frac{\partial \Theta}{\partial t} \qquad (8)$$

where

$$\Theta = \frac{T - T_b}{T_o - T_b}$$

Initial condition:

$T_i = T_o$ i.e.

$\Theta = 1$ at $t = 0$ for all x and y (9)

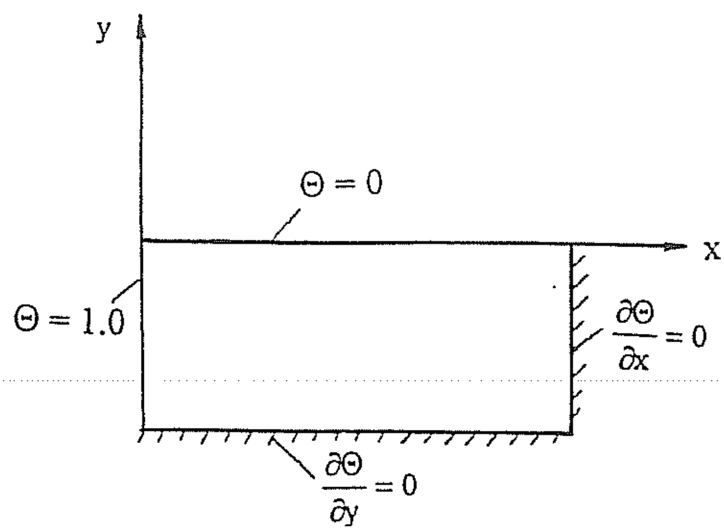

Fig. (3) Domain of a simplified problem
($\Theta = 1$ at $t = 0$)

Boundary conditions:

$$\frac{\partial \Theta}{\partial x} = 0 \qquad \text{at } x = L \qquad \text{for all y and t} \qquad (10)$$

$$\Theta = 0 \qquad \text{at } y = 0 \qquad \text{for all x and t} \qquad (11)$$

$$\Theta = 1 \qquad \text{at } x = 0 \qquad \text{for all y and t} \qquad (12)$$

$$\frac{\partial \Theta}{\partial y} = 0 \qquad \text{at } y = -H \qquad \text{for all x and t} \qquad (13)$$

The analytical solution of Eq. (8) subjected to boundary conditions, Eq.(9) - (13) is as follows [15,16]:

$$\Theta = \frac{2}{H}\sum_{n=0}^{\infty} \frac{1}{\Gamma_n \cosh(\Gamma_n L)} \cosh(\Gamma_n (L - x) \sin(\Gamma_n y)$$

$$+\frac{4}{HL}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \left(\frac{\Gamma_n^2}{\Gamma_n^2 + \beta_m^2}\right)\frac{1}{\Gamma_n \beta_m} \sin(\beta_m x).\sin(\Gamma_n y)\exp(-\alpha(\beta_m^2 + \Gamma_n^2)t \qquad (14)$$

where the eignvalues are given by:

$$\Gamma_n = \frac{2n+1}{2H}\pi, \qquad n = 0,\ 1,\ 2,\ \ldots\ldots$$

$$\beta_m = \frac{2m+1}{2L}\pi, \qquad m = 0,\ 1,\ 2,\ \ldots\ldots$$

Table (1) Thermophysical properties of different media [17], $\Phi = 0.4$

property	sand stone	wet soil 10 % moisture	dry soil	stainless steel	water	air
k W/m K	1.85	1.22	0.52	17.4	0.9	0.025
ρ kg/m^3	2225	1550	2050	7400	999	1.205
c J/kg k	710	710	1840	5016	4180	1006

5-RESULTS AND DISCUSSIONS:

The results of the present study are compared with those obtained from the analytical solution given by Eq. (14). This solution is available only for a simplified problem, i.e. rectangular region. This region is assumed homogeneous and its properties are taken for wet soil (10% moisture) given in Table (1). For purpose of verification, the initial and boundary conditions are chosen to be equivalent to those used in modeling as shown in Fig. (3). The variation of the dimensionless temperature with the dimensionless depth in the soil for various times up to 15 days is indicated at two different surfaces. One is at the middle of the pipe length, Fig. (4), and the other is at its end, Fig. (5). The results of the proposed model are in good agreement with the analytical solution.

In this analysis the results are presented for three different cases applied on a buried pipe made of stainless steel of 0.6 m inner diameter and 0.72 m outer diameter. The pipe carries water from a constant temperature reservoir at 3 °C, which is considered to be a suitable temperature to keep water in the pipe in liquid phase. . The flow in the pipe is considered laminar. Initially, the temperature in all media are 3 °C. The temperature at the surface of the soil suddenly decreases to a temperature less than zero and remains constant at this value. Four different values of surface temperature are considered mainly -5, -10, -25 and -36 °C respectively. In the first three cases the surface temperature is chosen to be -36 °C which corresponds to expected severe cold condition, while the fourth case illustrates the effect of variation of surface temperature on the working length. The thermophysical properties of the different media used in the cases studied are given in Table (1). In all cases, when the temperature of the top surface of the pipe reaches zero the water becomes at the freezing condition. This situation is considered as a criterion for the working conditions. The corresponding length up to the freezing location is called here the working length of the pipe.

Case (1):

In this case the effect of the burial depth of the pipe on its working length is studied at flow Reynolds number of 1100. The variation of the working length with time for different burial depth in sand stone is shown in Fig. (6). It is worth to notice that for all values of burial depth as the working length increases the time up to start of freezing of water inside the pipe decreases. This is obvious close to the inlet of the

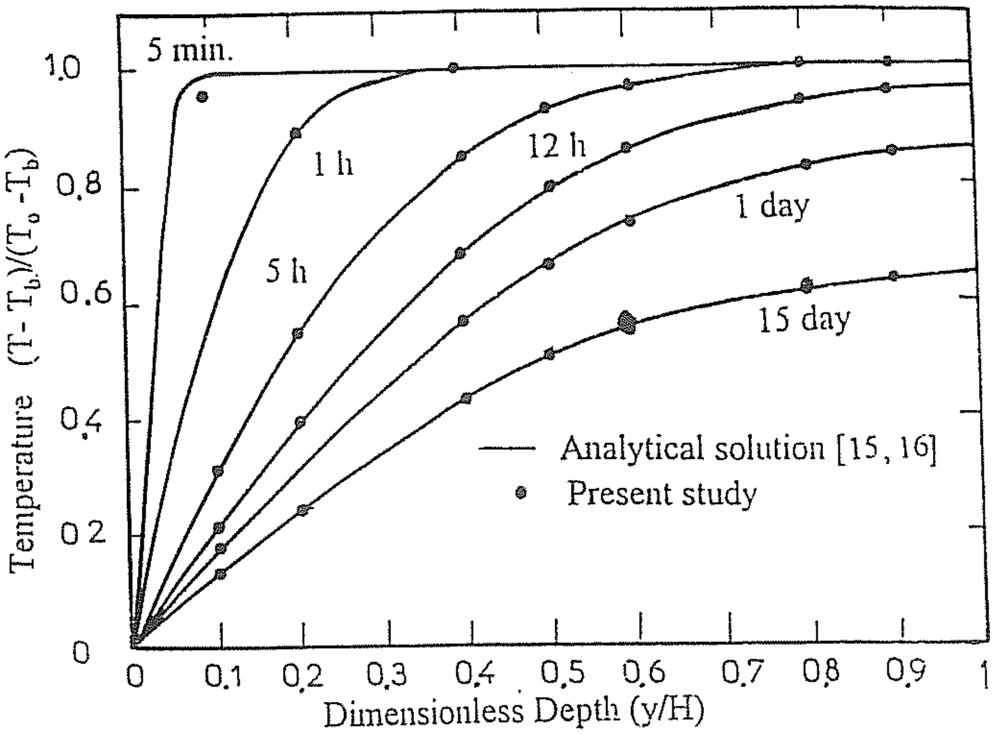

Fig. (4) Analytical and numerical solutions of a simplified problem $(T-T_b)/(T_o-T_b)$ versus depth, at x = L/2

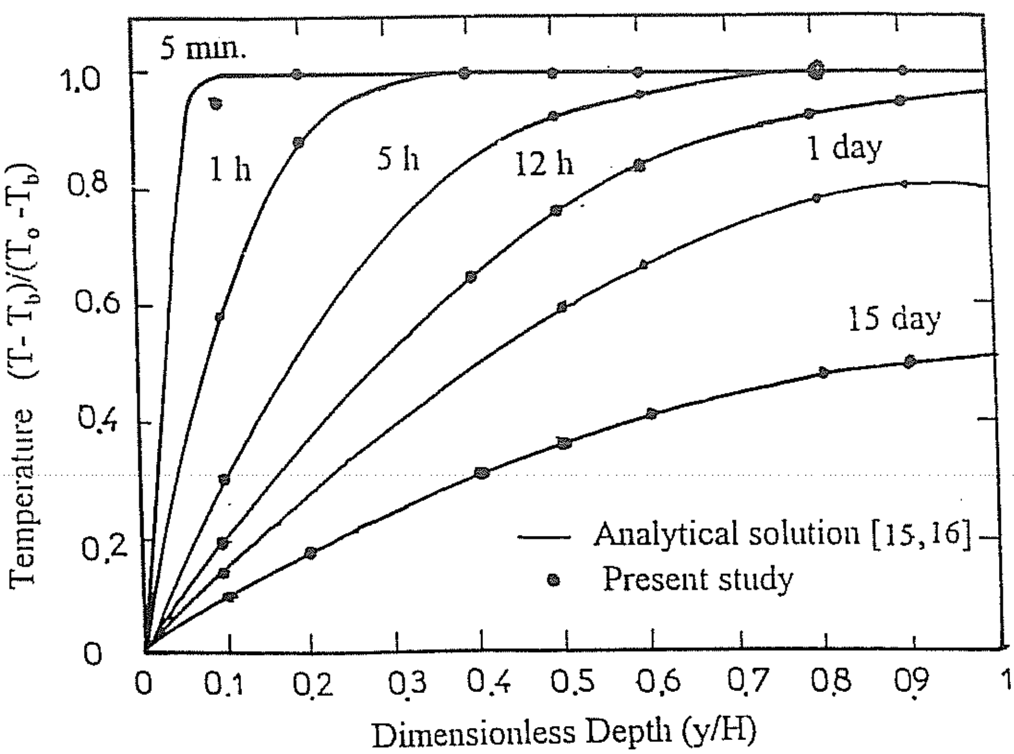

Fig. (5) Analytical and numerical solutions of a simplified problem $(T-T_b)/(T_o-T_b)$ versus depth, at x = L pipe. As the working length increases further, the time remains unchanged. This means that steady state condition is reached. Figure (7) shows the variation of the working length of the pipe with time for different burial depth in wet soil. Similar behaviour is noticed but the corresponding time up to freezing increases. A considerable increase in the working length and the corresponding time is noticed when the pipe is buried in dry soil as indicated in Fig. (8).

Case (2):

The effect of flow characteristics on the location of freezing of water inside the pipe, buried in the same soil (wet soil) is shown in Fig. (9). It is obvious that as Reynolds number increases the working length increases for all burial depth. In other words for higher values of Reynolds number and same working length the time up to freezing increases due to the heat coming with the flow.

Case (3):

This case indicates the effect of using different soils in which the pipe is buried on the working length for the same Reynolds number. Figure (10) shows the variation of working length with burial depth for three different soils. It can be seen that the dry soil give the greatest length for all burial depth, while the sand stone shows the smallest values of working length. It can be concluded that as the thermal diffusivity of the soil decreases the heat loss from the flowing fluid through the pipe decreases, consequently the corresponding working length of the pipe increases.

Case 4:

Figures (11), (12), and (13) show the variation of the working length of the pipe with time for various burial depths in wet soil at the surface temperatures of -5, -10, and -25 °C respectively. It can be noticed that as the surface temperature decreases the working length of the pipe decreases, i.e. the heat loss from the fluid flowing through the pipe increases.

Figures (14) and (15) show the isothermal contour lines at a burial depth 0.9 m in a wet soil, 90 m long pipe and Reynolds number 1100 for 15 h and 30.4 h, respectively The freezing temperature of the water is reached at the top surface of the pipe at 30.4 h. It can be noticed that the higher thermal conductivity of the metal

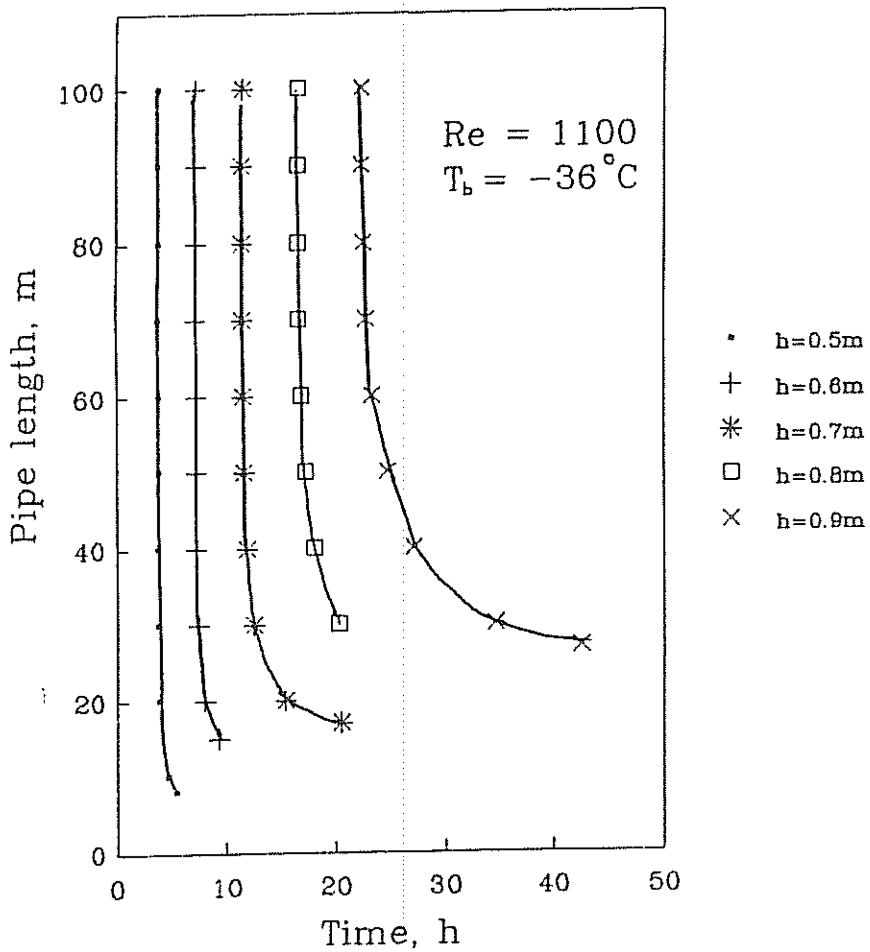

Fig. (6) Variation of the working length of the pipe with time for various values of burial depth in sand stone

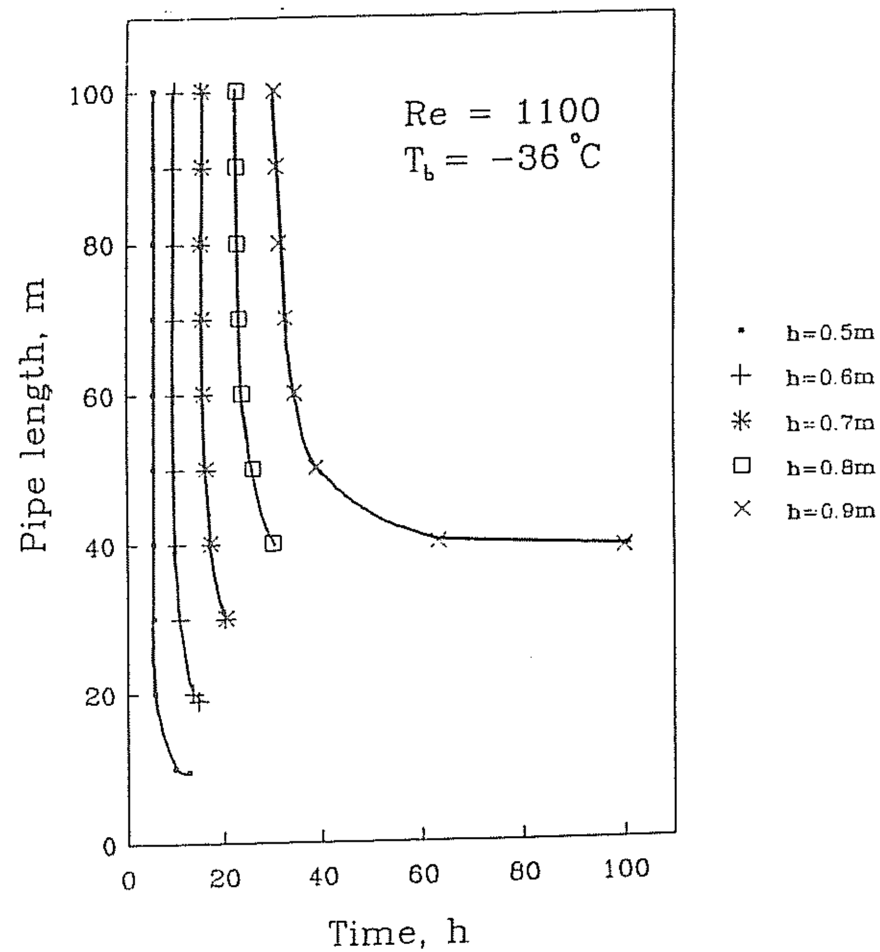

Fig. (7) Variation of the working length of the pipe with time for various values of burial depth in wet soil

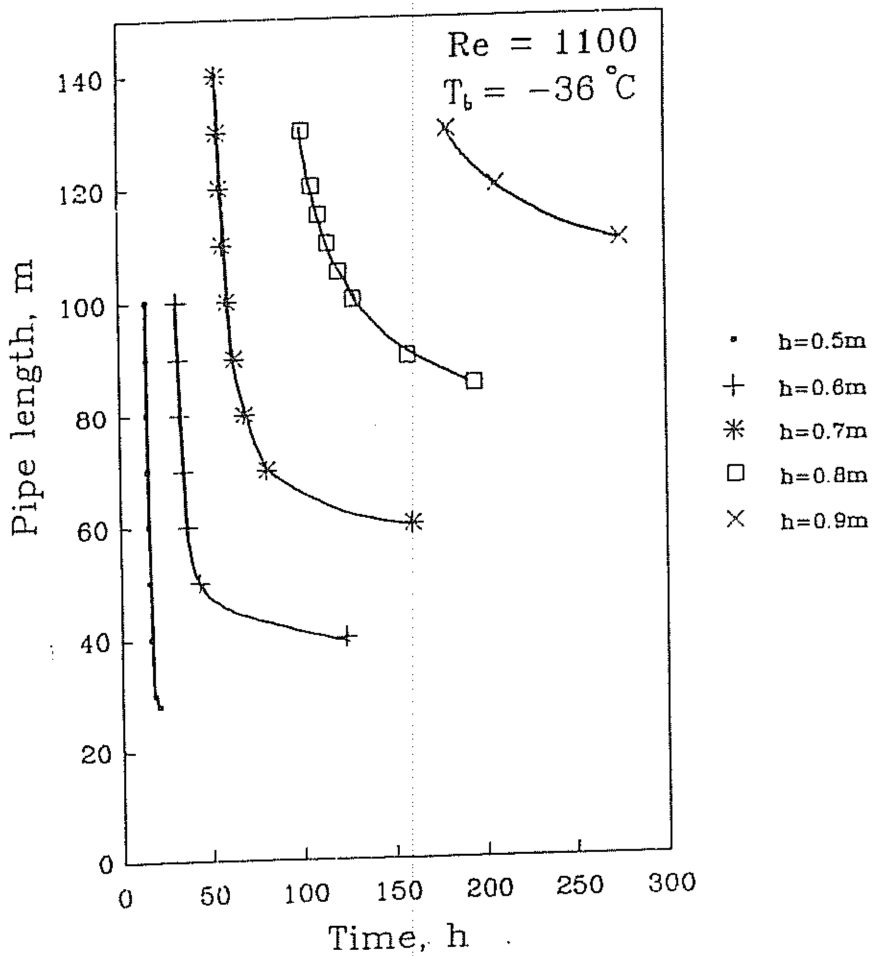

Fig. (8) Variation of the working length of the pipe with time for various burial depths in dry soil

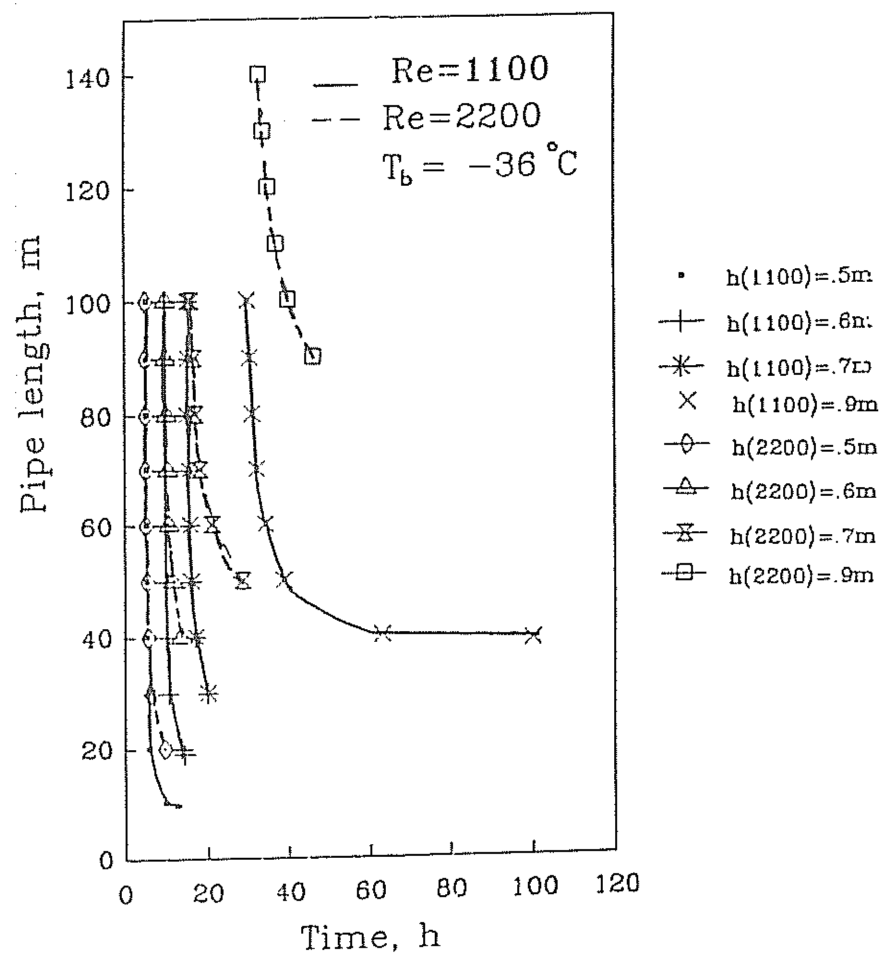

Fig. (9) Variation of the working length of the pipe with time for various burial depths in wet soil for various Reynolds number

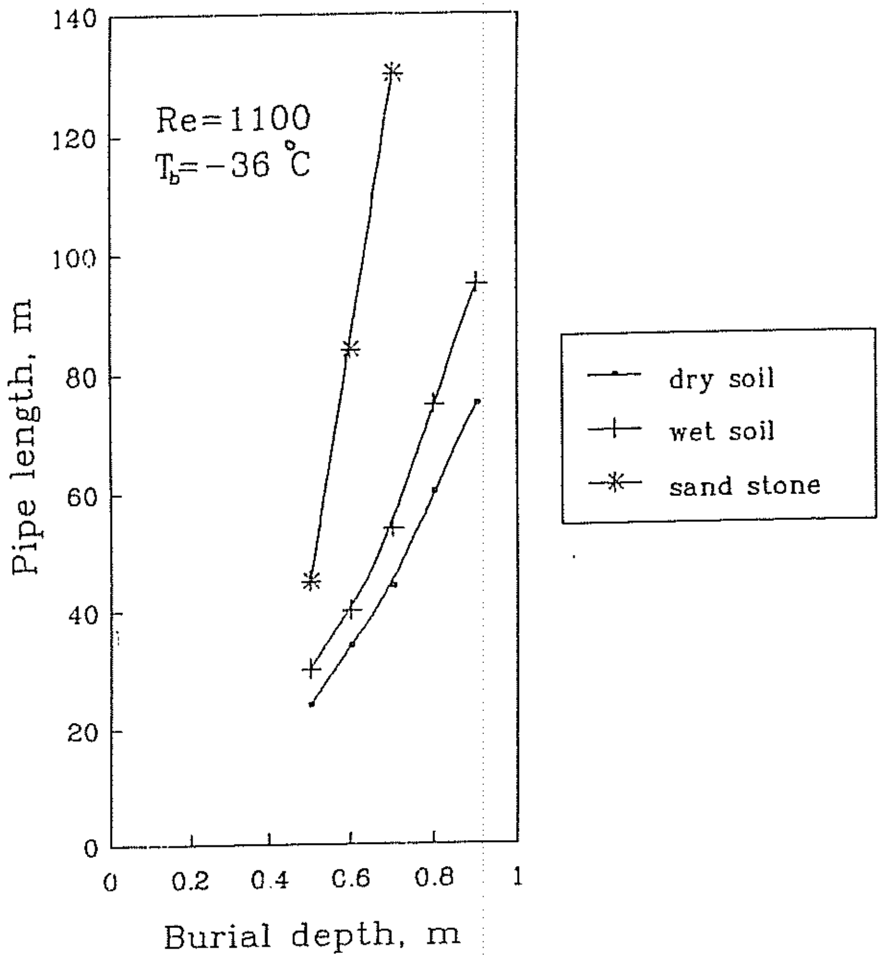

Fig. (10) Variation of the working length of the pipe with the burial depth for different soils

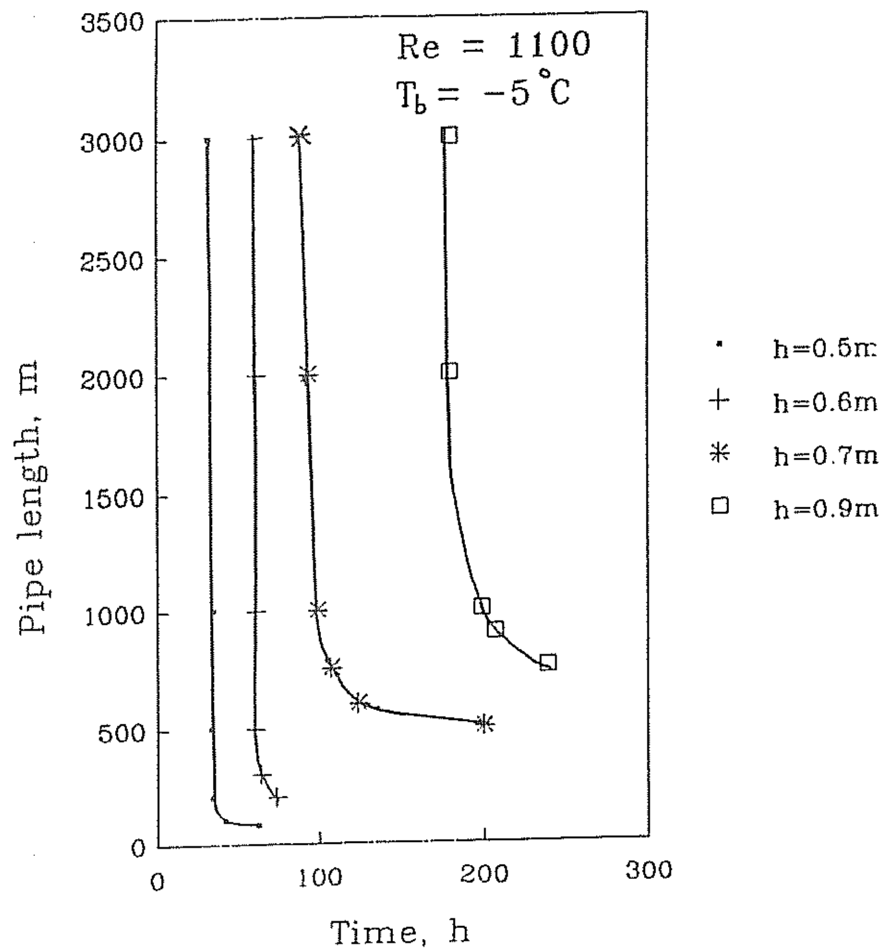

Fig. (11) Variation of the working length of the pipe with time for various burial depths in wet soil

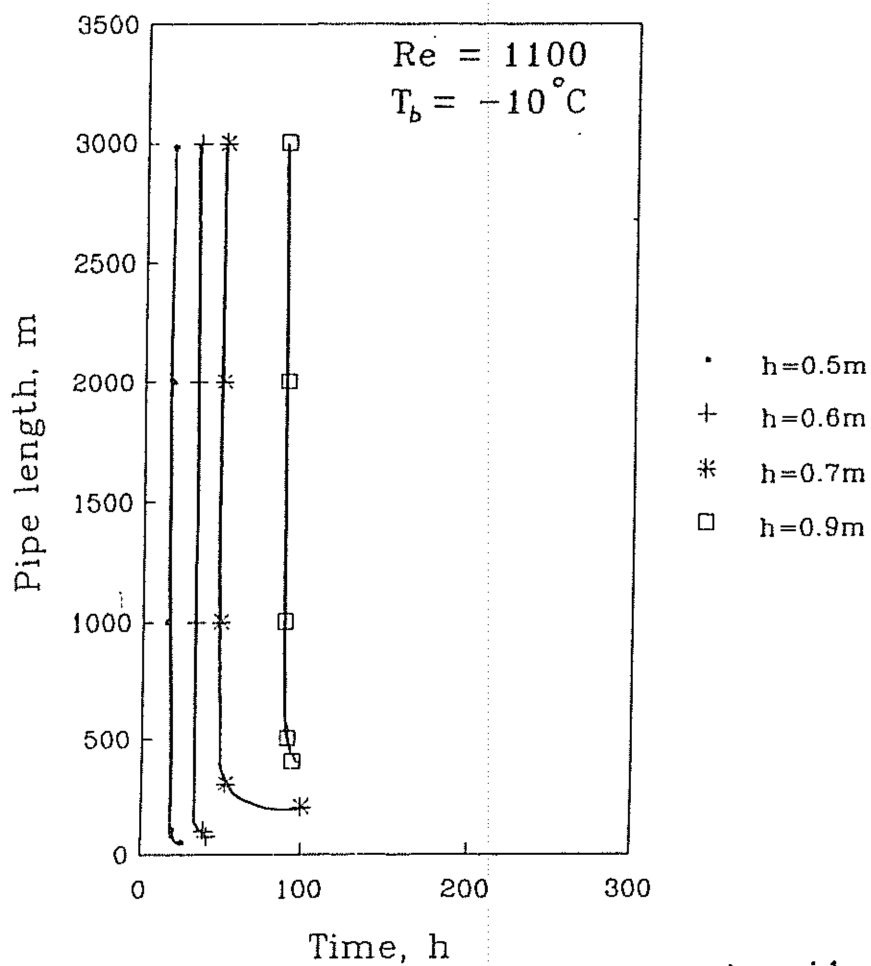

Fig. (12) Variation of the working length of the pipe with time for various burial depths in wet soil

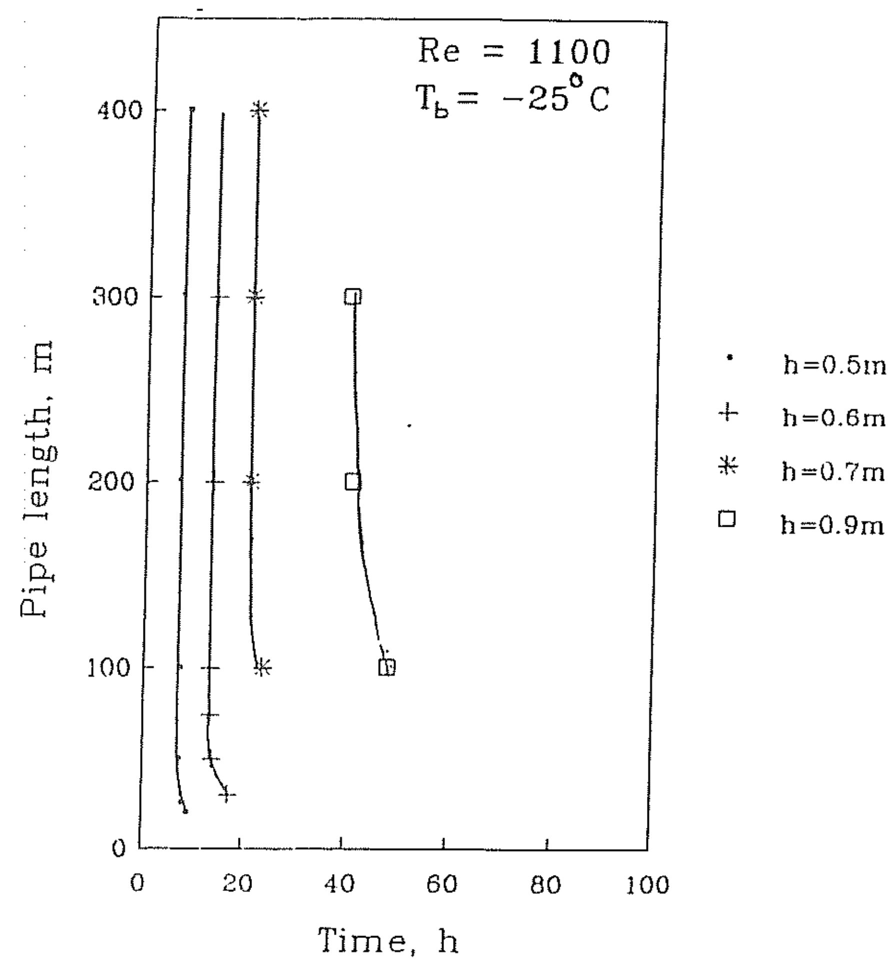

Fig. (13) Variation of the working length of the pipe with time for various burial depth in wet soil

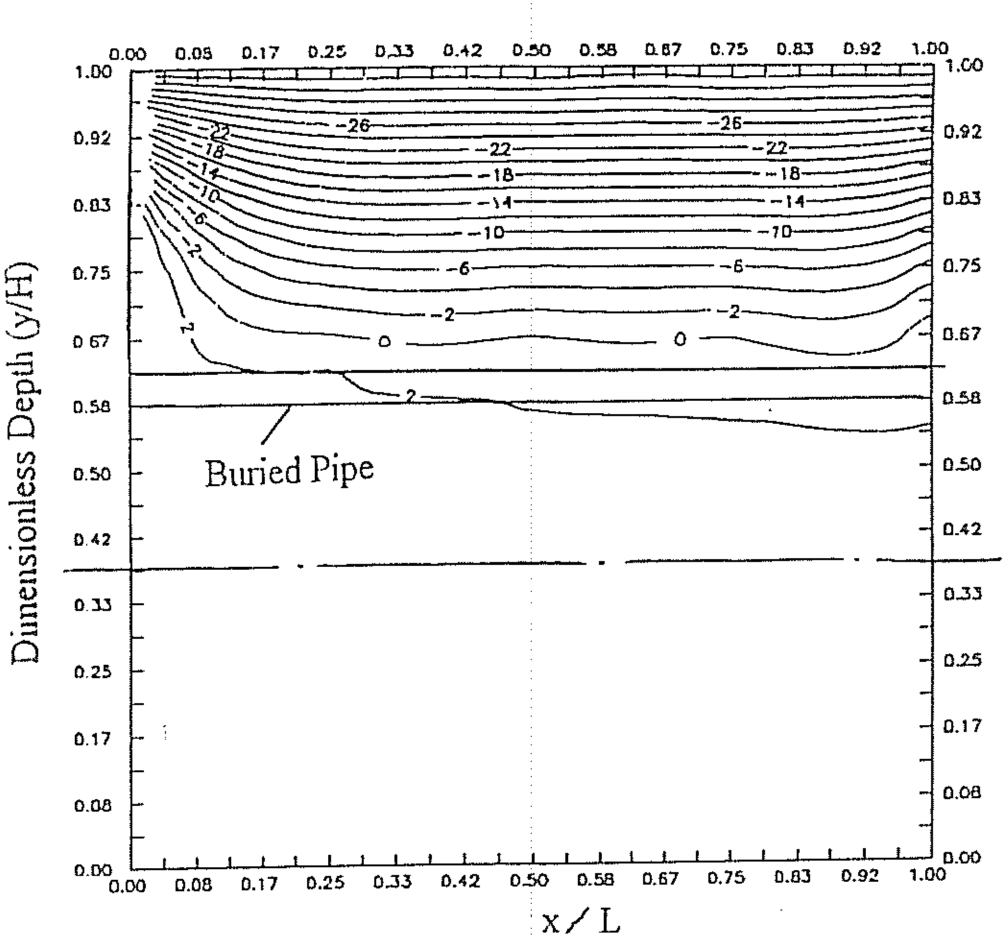

Fig. (14) Isothermal contour lines in wet soil at 15 h, Re=1100, L=90 m, h=0.9m

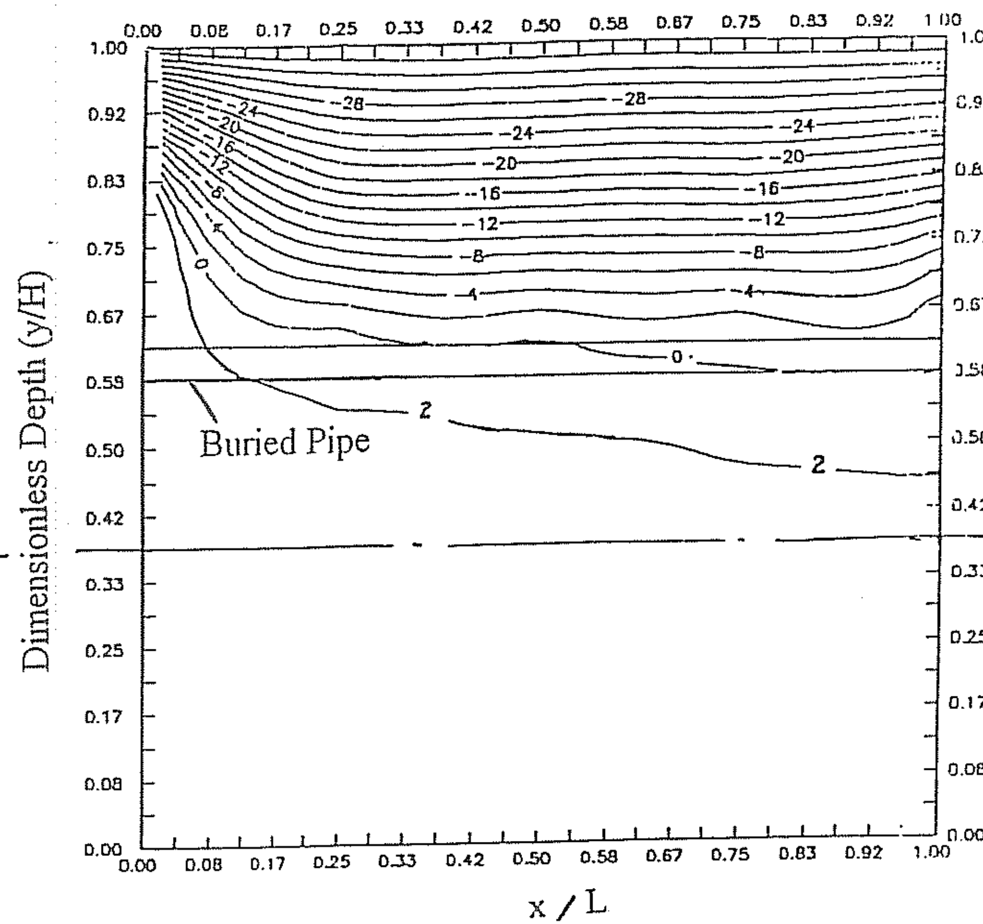

Fig. (15) Isothermal contour lines in wet soil at 30.4 h, Re=1100, L=90 m, h=0.9m affects the gradient of the isothermal contour lines markedly as they cross the pipe wall.

6-CONCLUSIONS:

A two-dimensional numerical model is developed to simulate the transient heat transfer from fluid flowing through buried pipes. The model is formulated using the finite elements technique. The model is developed in a general way that more complicated boundary conditions, the non homogeneous media, and wide range of flow characteristics can easily be incorporated. The results are obtained for different values of burial depth, flow velocity and soil properties. The results indicate that the maximum working length (corresponding to the freezing of the water at the top surface of pipe at end of this length) increases with:

1- the increase of the depth in which the pipe is buried
2- the increase of the flow velocity (Reynolds number)
3- using soil having small thermal diffusivity, e.g. dry soil.

The importance of this analysis is clear in that it can be used as a guide in determining the most suitable variables to achieve a desired length of the pipe inside which the flowing fluid remains liquid under severe cold environmental conditions.

NOMENCLATURE

A	area of triangular element, m^2
c	specific heat, J/kg K
c_p	specific heat at constant pressure, J/kg K
D	pipe inner diameter, m
H	total depth of the soil in y direction, m
h	distance between the pipe centerline and the top of the soil, m
{F}	nodal force vector
[K]	stiffness matrix
$[K_c]$	capacity matrix
k	thermal conductivity, W/m K
L	length of the pipe, m
N	interpolation function
n	unit vector

Re	Reynolds' number = $\rho uD/\mu$
R_i	inner pipe diameter, m
R_o	outer pipe diameter, m
T	temperature, °C
T_b	temperature at the top surface of the soil, °C
T_i	initial temperature, °C
T_o	temperature at the inlet surface, °C
t	time, s
U	average flow velocity of the fluid in the pipe, m/s
u	axial velocity, m/s
x, y	Cartesian coordinates, m

Greek

α	thermal diffussivity, $k/\rho c$
β_m, Γ_n	eignvalues of the simple problem
Θ	dimensionless temperature, simplified problem
μ	viscosity, $N.s/m^2$
ρ	density, kg/m^3
Φ	porosity

Superscripts

e	element
.	derivative w.r.t time

Subscripts

a	air
es	effective value of the soil
f	fluid
p	pipe
s	soil

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